On square integrable solutions and principal solutions for linear Hamiltonian systems

In the study of qualitative properties of linear differential, difference, and dynamic equations or systems we can identify, among others, two substantial directions: the oscillation theory and the spectral theory. In the first one, roughly speaking, the numbers of (generalized) zeros of real-valued solutions are investigated and a crucial role is played by the *principal solution*, which existence is equivalent to the nonoscillatory behavior of the equation. In the second one, associated operators are investigated and a crucial role is played by square integrable/summable complex-valued solutions, which numbers correspond to the deficiency indices. Especially, the square integrability/summability of the *Weyl solution* yields a lower bound for these numbers. In this talk, we show that there exists a close connection between the Weyl solution and the principal solution of the linear Hamiltonian differential systems

$$z'(t,\lambda) = [\mathcal{H}(t) + \lambda\, \mathcal{J}\, \mathcal{W}(t)]\, z(t,\lambda), \quad \mathcal{J} := \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}, \tag{H_λ}$$

where $t \in [a,\infty)$, $\lambda \in \mathbb{C}$ is a spectral parameter, and $\mathcal{H}(t)$ and $\mathcal{W}(t)$ are piecewise continuous even order matrix-valued functions such that $\mathcal{J}\mathcal{H}(t) + \mathcal{H}^*(t)\mathcal{J} = 0$ and $\mathcal{W}(t) = \mathcal{W}^*(t) \geq 0$ for all $t \in [a,\infty)$; compare with [1, Theorems 2.13 and 3.11] for the second order Sturm–Liouville differential equations. In particular, we will see that the property of "being the Weyl solution for $\lambda \in \mathbb{R}$" is more general that the property of "being the minimal principal solution at infinity". Furthermore, we present also another results in the Weyl–Titchmarsh theory for system (H_λ), which were derived by using principal solutions.

The talk is based on the joint research with R. Šimon Hilscher, see [2].

References

[1] S. L. Clark, F. Gesztesy, and R. Nichols, *Principal solutions revisited*, in "Stochastic and Infinite Dimensional Analysis", C. C. Bernido, M. V. Carpio-Bernido, M. Grothaus, T. Kuna, M. J. Oliveira, and J. L. da Silva (editors), Trends in Mathematics, pp. 85–117, Birkhäuser, Basel, 2016.

[2] R. Šimon Hilscher and P. Zemánek, *On square integrable solutions and principal and antiprincipal solutions for linear Hamiltonian systems*, Ann. Mat. Pura Appl. (4) (to appear).