1.

a. Two particles traveling along the space curves \( \mathbf{r}_1(t) = \langle t, t^2, t^3 \rangle \) and \( \mathbf{r}_2(t) = \langle 1 + 2t, 1 + 6t, 1 + 14t \rangle \). How many times (if any) do the curves intersect? In answering the question, either state that the curves never intersect or else find the point(s) of intersection.

b. Find a vector equation for the line segment between \( P(1, -1, 2) \) and \( Q(4, 1, 7) \).

2.

a. Find the unit tangent vector for \( \mathbf{r}(t) = \langle e^t, \cos t \cdot \tan t, \ln(4t + 1) \rangle \) at \( (1, 0, 0) \)

b. Evaluate \( \int \left( \frac{4}{3t + 5} \mathbf{i} + t \sin(3t) \mathbf{j} - \frac{e^t + t}{\pi - 1} \mathbf{k} \right) dt \)
3. a. Find the length of the curve \( \mathbf{r}(t) = \left( \sin \left( \frac{1}{2} t \right), \cos \left( \frac{1}{2} t \right), 1 \right), \) \( 0 \leq t \leq 2\pi \)

b. Find the curvature of \( \mathbf{r}(t) = (2t + 5)i - tj + \left( \frac{t^2 - 1}{t - 1} \right)k \) at \( t = 14 \).

4. A particle starts at the top of a 500 meter cliff with initial velocity \( i - j \). Its acceleration is due solely to gravity. Find its position function using the technique presented in Chapter 14.

5. a. Find and describe the domain of \( f(x, y, z) = \sqrt{1 - x^2 - y^2 - z^2} \)

b. Describe with equation and words the level surfaces of the function \( f(x, y, z) = x^2 + 3y^2 + 5z^2 \) for:

   \( k = 0 \):

   \( k > 0 \):
6. a. The function \( f(x, y) = \begin{cases} \frac{x^2 - y^2}{x^2 + y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases} \) has a discontinuity at \((0, 0)\). 
   Show why this is so.

b. Often times, we use the Squeeze Theorem to evaluate the limit of a multivariate function. Explain how the Squeeze Theorem works.

7. Find all first partial derivatives:
   a. \( z = f(x/y) \)
   b. \( z = f(x)g(y) \)
   c. Consider the function \( p = f(x, y, z, w, q) \) and its 4\(^{th}\) degree higher-order partial derivative \( \frac{\partial^4 p}{\partial w \partial x \partial q \partial z} \). According to this notation, which independent variable would you differentiate against first? (circle all that apply)

   \[ p \quad \text{same as } f_{wxyz} \quad \text{none of these} \]
8. a. Find an equation of the tangent plane to the surface \( f(x, y) = \sqrt{xy} \) at the point (3,4).

b. What is the difference, if any, between the differential \( dz \) and the increment \( \Delta z \)?

9. a. Suppose \( f \) is a differentiable function of \( x \) and \( y \), where \( g(u, v) = f(e^u \sin v, e^u \cos v) \). Find \( g_u(0,0) \) given the following information:

\[
f(0,0) = 3, \quad g(0,0) = 6, \quad f_x(0,0) = 4, \quad f_y(0,0) = 8
\]
\[
f(1,2) = 6, \quad g(1,2) = 3, \quad f_x(1,2) = 2, \quad f_y(1,2) = 5
\]

10. a. Find the rate of change of \( f(x, y) = ye^{-x} \) at the point (0,4) in the direction of

\[
\theta = \frac{2\pi}{3}
\]

b. What is the significance of the gradient of a function?