Math 22 Exam 2C

Akers — FS 00

1) (10 pts) Given $f(x,y) = xy + e^y \cos y - 2$ find f_x, f_y, f_{xx}, f_{yy} and f_{xy}.

2.(5 pts) Given $z^3 - zy = \cos x + 4$, find $\dfrac{\partial z}{\partial x}$.

3.(10 pts) **Use the Chain Rule** to find $\dfrac{\partial w}{\partial s}$ when s=1 and t=0 if

$w = \sqrt{x} + y^2 + z$ and $x = e^{2t} + s^2$, $y = t^2 + 1$, $z = s(t-1)$.

4.(10 pts) Find the equations of the tangent plane and the normal line to the surface $z^2 + zx - y^2 = 2$ at the point $(1,2,-2)$.

5a. (10 pts) Find the rate of change of $z = f(x,y) = 2\sqrt{x} - y^2$ at the point (1,5) in the direction of $\langle -3,4 \rangle$.

5b. (5 pts) In 5a. above, at the point (1,5), what is the **direction** of the maximum rate of change of the function and what is the **magnitude** of this maximum rate of change?

6. (10 pts) Show the following limit does not exist and clearly show why.

$$\lim_{(x,y)\to(0,0)} \frac{2x^2y}{x^4 + y^2}$$

7. (10 pts) Given $z = f(x,y) = y\sqrt{x}$, find the linearization $L(x,y)$ for $f(x,y)$ at the point (4,1) and use it to approximate the value of $(1.1)\sqrt{3.92}$. Compare your approximation with your calculator's value.

$L(x,y)$ =_________________

8. (15 pts) Given $f(x,y) = \frac{1}{2}x^2 + y^2 + \frac{1}{2}x^2 y + 7$, find and classify all critical points of $f(x,y)$.

9. (10 pts) A company makes widgets and thingamajigs and their monthly profit is modeled by $P(x,y) = x^2 y$, where x is the number of widgets and y is the number of thingamajigs. Use Lagrange multipliers to find the values of x and y that will maximize the company's profit if the total number of items the company can make in a month is 90. (i.e. $x + y = 90$)

10. (5 pts) You are standing on **top** of a hill at an elevation of 300 feet. The terrain falls away from you in all directions. If you consider the hill to be defined by $z = f(x,y)$, what is the gradient vector, ∇f, where you are standing? If you next consider the hill to be defined by $F(x,y,z) = f(x,y) - z = 0$, what is the direction of ∇F where you are standing?

∇f =_____________

direction of ∇F =_____________